Article

A Lesson Study on Using the Concrete-Pictorial-Abstract (CPA): Approach in Addressing Misconceptions in Learning Fractions

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Abstract

This paper explores the use of the Concrete-Pictorial-Abstract (CPA) approach in teaching operations on fractions to address the misconceptions in learning fractions. Through a lesson study, the researchers aimed to reflect on and evaluate the effect of the planned activities on students' engagement and processing of the learning of the research topic. The study was conducted in an elementary school in Las Piñas City, Philippines, and the various results and recommendations were as follows: (1) providing concrete examples in the context of the students enhances students' engagement, (2) stimulating and processing responses from students help develop the concept, and (3) connecting the learning to various subjects or fields deepens the students' understanding of the topic.

Keywords: Concrete-Pictorial-Abstract (CPA) Approach, Operations on fractions, lesson study

Introduction

Concepts underlying fractions have always been one of the greatest challenges to teaching and learning in the network of mathematical education. These challenges right off the bat in essential years (Brown & Quinn, 2006), then into secondary and even tertiary education (Kamii & Clark,1995). The challenges and misunderstandings students face in understanding fractions (Cantoria, 2016) continuously exist even during adult life which affect different areas in such wide-ranging fields as medicine and health care, construction, and computer programming. With this, the fields of science, technology, engineering, and mathematics (STEM) request impressive and in-depth knowledge of fractions; a weak foundation in this skill can inhibit individuals from pursuing mathematics at its superior level, may affect the choices of students in choosing their career, or deprive students of some career opportunities in the future.

The International Bureau of Education (IBE) of UNESCO pointed out that oftentimes the reason why students have some difficulty learning fractions is the lack of conceptual understanding. Understanding the concept of the numerator and denominator is some of the key concepts in fractions. However, there are a lot of students who still view them as two separate numbers rather than a unified whole. As cited in the research by Beswick & Jennison (2009), the topic of rational numbers especially those which involve fractions is complex. Many high school students could not or have difficulty grasping the concepts, which may be the reason why some teachers are not confident in teaching the topic as well. As Van de Walle & Lovin (2006) said, this is the area where most students give up on trying the underlying concepts but just rely on the different rules given per se.

Some issues arise within mathematics educators regarding the methodology for teaching fractions to students since many have a lot of differences in terms of understanding the concept. In the study by McLeod (1992), he concluded that it is important for teachers to be aware of both the cognitive and affective aspects of learning and teaching mathematics. Fazio and Siegler (2011) also agreed with this conclusion and stated that by cultivating conceptual understanding, teachers can help students to understand that fractions are real numbers, and that fraction arithmetic is a meaningful procedure rather than a series of arbitrary steps. Conceptual understanding is difficult to acquire, but it is vital for ensuring a deep and enduring understanding of fractions and fraction arithmetic.

In the Philippine basic education curriculum based on the DepEd K-12 Curriculum Guide for Mathematics, rational numbers or fractions are taught as early as Grade I. The curriculum is arranged in such a way that students' visualization, modeling, and the representation of unit fractions with even denominators in first grade to a more advanced evaluation of complex rational expressions in high school algebra are developed. However, Cantoria (2016) cited that regardless of how the underlying concepts, principles, and operations of fractions recur in the curriculum, many high school graduates still reach tertiary level without mastery.

Designing Lessons to Address Misconceptions in Learning Fractions

Bruner (1966) stated that learning should be inductively done. Also, he added that a theory of instruction should address aspects that include structures that will help learners easily grasp the lesson, and sequences that will let the students go beyond the information given. With this, the proposal for the three stages of learning namely, enactive, iconic, and symbolic, were created. Through this idea, the formulation of the Concrete-Pictorial-Abstract (CPA) approach was invented.

At the enactive level, the child manipulates materials directly. This is commonly referred to as the "concrete" stage. Bruner believed that "learning begins with an action - touching, feeling, and manipulating" (Brahier, 2009, p. 52). In the CPA approach, this is the "doing" part because it involves physical manipulation of objects (i. e., fraction kits, geoboards, etc.) to solve a problem. Furthermore, the iconic level is where the student deals with mental images of objects, but the student does not manipulate them directly. This stage represents the "pictorial" part, which involves the representation of the connections evident in the first step. One way of doing this is by using shapes, diagrams, and graphs. Finally, "abstraction" happens at the

symbolic level. Manipulating symbols is used to model problems and demonstrate understanding of the task.

In this paper, the CPA approach is explored in teaching the operations on fractions to address the misconceptions about learning them. Lesson study is used as a methodology for analyzing lessons (Elipane & Bonus, 2015) that makes use of the said approach.

Methodology

To explore the use of the Concrete-Pictorial-Abstract (CPA) approach to teaching operations on fractions in addressing the misconceptions about learning fractions, the group used lesson study to reflect on the planned activities.

Lesson study is a professional development model or process that originated in Japan (Elipane, 2015). It allows the teacher to collaboratively learn to improve the quality of classroom pedagogy. The researchers followed certain steps to achieve our goal, which involved the following: 1. planning the research lesson; 2. performing the research lesson; and 3. reflecting on the outcomes of the research lesson. The group also invited the teachers at the school where they conducted the lesson study and other guests who had prior knowledge or background on how lesson studies are done.

a. Planning the Research Lesson

Operations on fractions were chosen because they are the basis of more complex topics in mathematics such as algebra, geometry, and trigonometry. The group focused on how to improve students' learning, such as making the students understand the concepts behind each topic, and how to make sense of this learning. In the K–12 Curriculum Guide for Mathematics, fractions should be introduced in first grade and the first quarter of sixth-grade mathematics must focus on the mastery of the operations on fractions and decimals. (See Table 1).

Table 1
K–12 Curriculum Guide of Mathematics on Fractions

CONTENT		
Numbers and Number Sense		
CONTENT STANDARDS	**PERFORMANCE STANDARDS**	**LEARNING COMPETENCIES**
The learner…	The learner…	The learner…
demonstrates mastery of the operations of fractions and decimals.	*is able to add and subtract fractions and decimal numbers and solve problems involving these numbers.*	*adds and subtracts fractions in simple mixed forms without or with regrouping* *solves problems involving addition and subtraction of fractions and decimals in simple or mixed forms without and with regrouping.*

The researchers carefully formulated their word problem. The problem encompassed some basic topics, like "Addition and Subtraction of Fractions", "Changing Mixed Numbers to Improper Fractions", and "Area and Perimeter". It is assumed that students learned it multiple times, but with varying levels of difficulty, since the Philippines is adapting to a spiral curriculum. The problem was designed to be answered only by "*yes" or "no",* but the students were asked to discuss their answers afterward for the class to know the reason for their answers. The problem would serve as their anchor task or goal to be accomplished after the discussion and activities carefully prepared by the teacher.

The group was able to invite twenty-five (25) students from an elementary school in Las Piñas, Philippines. The lesson study took place at that school, wherein each section was represented by a student randomly chosen by their respective mathematics teachers.

b. Implementing the Research Lesson

The lesson started with an activity called "I'm incomplete: Show Me My Piece". The teacher distributed fraction kits to every student and started the activity by asking the students to pick one strip of paper. This activity is aimed at conditioning the minds of the students that each strip represents a whole, "*This is one whole, show me one – half*.". As was observed, the students showed their unique way of showing "*one–half*" to their teacher. Continuously, the teacher provided questions about the relationship between the parts of the whole by asking the students to show "*one – third, one – fourth, one – fifth, one-sixth and one – eight*."

The activity was used not only to stimulate the children's interest in learning fractions, but to help the students realize that fractions are not just made-up numbers. Since these materials prepared are tangible for them to manipulate, they can self-actualize that there are fractions in real life.

After the initial activity, the students were grouped randomly and were given a problem. One of the goals in answering the problem is to decide whether to provide an additional fence on the new rectangular fence based on the perimeter of an originally square fence.

Results and Discussions

Before discussing different suggestions of the research group on how to develop a good lesson on the teaching of fractions based on the conducted lesson study, we will go through first the different misconceptions and cultures of the students.

1. In addition, and subtraction of fractions, students tend to add both numerators and denominators to get the answer.

One group almost had the correct answer as they were able to represent the problem using pictorial representation. They were able to put together the correct number sentences from them, but while they were performing the operations, they added both the values of the numerator and the denominator, leading to an incorrect answer.

2. When they were solving word problems, the students tried to find *keywords* like "sum, difference, total, etc." to identify what operation should be used.

An observation that we have made while listening to the conversation of the groups while working is that they tend to find keywords to help them identify operations to be used. One student said, "*May word na add sa problem, kaya addition ang gagamitin natin,"* (There's the word "adding" in the problem so we have to use addition to solve it.) Another student in the same group argued, "*Pero may reduce na word rin, 'di ba subtraction yun?* (But there is a word "reducing"; it suggests subtraction, right?). With this conversation, the groups' output was that they added the first two fractions, and the sum was decreased by the third fraction given.

3. In working with addition and subtraction involving mixed numbers, students tend to convert the mixed numbers into improper fractions first before combining them.

Although addition and subtraction of mixed numbers could be solved this way as well, what we have noticed is that sometimes, because of too many steps that they need to follow, students can be confused about how to do it. Moreover, they are doing these steps not because they see that improper fractions and mixed numbers are related to each other but because it is the process introduced to them by the teacher. When asked why needed to change them into improper fractions, they answered, "*E kasi ganon talaga."* (It is because that is how we do it.)

4. The students formed a habit of using the acronym "AGONA" to solve word problems.

AGONA stands for, **A**sked, **G**iven, **O**perations to be used, **N**umber Sentence, and **A**nswer. Most of the groups used AGONA to solve the problems given to them. It may suggest that this is the culture that they were used to in solving problems, as the teacher during the lesson study did not require the students to solve the problem nor suggest what strategy they should use.

The group has found three nascent factors that relate to scrutinizing how to improve the teaching of the operations of fractions and focusing on the misconceptions that were extracted from the Lesson Study: (1) providing concrete examples in the context of the students enhances students' engagement, (2) stimulating and processing responses from students helps develop the concept, and (3) connecting the learning to various subjects or fields deepens the students' understanding of the topic.

Providing Concrete Examples in the Context of Students Enhances Students' Engagement

The use of concrete examples so students can visualize the problem is one important thing to consider. The use of a fraction kit is helpful. However, it is also noticeable that even if the students are already in sixth grade, they still admit that the basic fraction skills are still not mastered. One observer overheard one student who said during the lesson that "*I still do not know how to add them*". It was also observed that during the time that the fraction kit was still being used, it was evident that the engagement of the students was to a high extent.

The result of our study agrees with previous studies which state that the fraction concept should be developed informally using a decent number of questions and with the same significant and reliable utilization of concrete materials (Empson & Levi, 2011). The researchers also found out that functional misconceptions under procedural knowledge, such as the inability to convert mixed numbers to an improper fraction and incorrect algorithms used in performing operations, can also be addressed through a smooth transition from concrete and pictorial models to abstract representations of fractions. This is present in the intervention (Elipane & Bonus, 2015) which significantly helped in correcting the students' misconceptions. This is supported by Piaget's theory (Ojose, 2008) and Mcguire's (2004) study. However, the observers found a "big leap" from the initial activity to the main activity because there were students who were not able to connect the functional knowledge to the problem that they were solving.

Stimulating and Processing Responses from Students Help Develop the Concept

During the initial exercise, the teacher used the fraction kit as a drill for the students. The students initially showed their way of depicting a half based on their fraction kit. As the lesson went along, there were different answers when the teacher asked the students to show him one–third. The teacher then asked the students how they arrived at that answer. The students who got it right were able to explain it and the students who were not able to do the task were guided when they solved additional problems.

Throughout the group activity, it has been observed that the students were sharing their ideas as they answered the questions. After the activity, each group had their representatives and explained their work in class. It was noticeable that the semantic misconceptions under conceptual understanding were present. It includes the inability to comprehend a given problem; analyze given statements in a problem and misinterpret words or phrases in a problem (Elipane & Bonus, 2015).

The students encountered a roadblock after presenting their activity wherein students made computational errors like adding fractions without finding a common denominator. It was addressed by demonstrating the need for adding similar units; this was done using the fraction kit prepared by the research group. The teacher asked the students to identify what 1/4 added to 2/4 with a fraction kit, and the students easily found out that the answer was 3/4 and showed it through counting the number of quarters in the manipulative that they were using. However, when the teacher asked the students to get the sum of 1/4 and 1/2, some of the students immediately answered 2/6. The teachers asked the students to show the result using the fraction kit wherein the students were able to represent both 1/2 and 1/4 as separated wholes. With some probing questions, the teacher led the students to generalize the procedure for combining unlike fractions.

Based on the observations, it would have been better if adding and subtracting similar fractions were addressed first before proceeding to the group activity. On the other hand, the observers commended one group that had shown a pictorial representation of the dimensions of the new rectangular fence, and if it had been focused on this group could have arrived at the correct answer to the problem.

Connecting the Learning to Various Subjects or Fields Deepens the Students' Understanding of the Topic

The observers agreed that the engagement of the students was evident, even if the activity used initially incorporated a concrete object. Also, this shows that collaboration can help towards the improvement of a lesson as the participants come with different experiences and can contribute.

It was observed that the students explained the connection between the operations on a fraction in their real life and they gave several examples of them, such as sharing their blessings with others.

Lastly, one observer commended the values integration part in the latter part of the lesson for being anchored in real-life and added that a real-life problem is solved by the precise concept of a fraction and that solving each problem carefully is important in our life.

Conclusions and Recommendations

It can be noted that using the CPA Method in discussing operations on fractions and addressing misconceptions about learning fractions was helpful. Using concrete examples and pictures tailored to their day-to-day lives, the students were able to easily exhaust all the necessary possibilities in a certain situation. By leading them to the abstract process, the students were able to make a connection with why they did such tedious processes earlier and observe how they could see all these possibilities more easily; thus, resulting in their persistence in trying to solve the problem on their own. Through this strategy, students can visualize and appreciate the essence of the lesson. Moreover, by using models in different fraction tasks, students can identify the concepts needed to know and discuss why we do certain processes in working fractions. They can own the experience of discovering their misconceptions and try to find ways to address them. It must be noted that in the entire process, the teacher served as a facilitator of knowledge, which resulted in students producing ideas on their own rather than just becoming receivers of knowledge.

It could also be concluded that planning the lessons and tasks for the students is very crucial. The research group only gave one problem for the class to solve—in line with this, all the reviews and tasks that were previously done before the group activity would be of help to the students in solving the anchored task. It was planned in such a way that it is not too leading for the students to be convened in only one method of solving the problem, nor too disconnected that the students may not be able to see any connections between the activities made and the tasks to be done.

With this research, it is recommended for future researchers to focus mainly on specific topics in solving problems involving fractions, particularly in addition and subtraction. As well, the knowledge before the lesson must be addressed. The researchers are raising one of the points of improvement, which is the consideration of the number of topics to be discussed with the students. It can be noted that the topics that will be discussed must have vivid application in the students' daily lives so they will be able to relate further to the discussion.

References

Almeda, D., Cruz, E., & Dy, A. (2013). *Addressing Students' Misconceptions and Developing Their Conceptual Understanding and Procedural Skills on Fractions Using Manipulative Materials*.

Brown, G., & Quinn, R. J. (2006). *Algebra students' difficulty with Fractions: An Error Analysis.*

Bruner, J. (1966). *Toward a Theory of Instruction*. Cambridge, MA: Harvard University Press. Retrieved August 2017, from http://www.instructionaldesign.org/theories/constructivist.html

Cantoria, Jr., A. L. (2016). *The predominance of Procedural Knowledge and Between-Operation Interference as Deduced from Fraction Errors of Preservice Teachers*. Asia Pacific Journal of Multidisciplinary Research, 75-79.

DepEd K-12 Curriculum Guide for Mathematics (2013). Retrieved August 2017, from http://www.deped.gov.ph/sites/default/files/Math%20Curriculum%20Guide%20Grades%201-10%20December%202013.pdf

Empson, S., & Levi, L. (2011). *Extending Children's Mathematics: Fractions and Decimals. Heinemann.*

Elipane, L. & Bonus, B. (2015). *Institutionalizing Lesson Study in teacher education institutions in the Philippines*. In Oh Nam Kwon (Ed), Proceedings of the Korean Society of Mathematical Education 2015 International Conference on Mathematics Education, Vol. 2 (pp. 155-163). Seoul: KSME.

Elipane, L. E. (2017). *Introducing Lesson Study as a Professional Development Model in the Islands of the Philippines.* Journal of Computational and Theoretical Nanoscience, 23 (2), 1126-1129.

Fazio, L. &Siegler, R. (2011). *Teaching Fractions*. Educational Practices Series-22. UNESCO International Bureau of Education. Geneva, Switzerland. http://www.ibe.unesco.org.

Gould, P., Outhred, L., & Mitchelmore, M. (2006). *One-third is three-quarters of one-half. MERGA*, 262-269.

Kamii, C., & Clark, F. B. (1995). *Equivalent fractions: Their difficulty and educational implications.* The Journal of Mathematics Behavior, 365-378.

National Center for Education Statistics (2005). *The Nation's Report Card.*

Orpwood, G., Schollen, L., Leek, G., Marinelli- Henriques, P., & Assiri, H. (2011). *College mathematics project 2010: Final report.* Toronto: Seneca College of Applied Arts and Technology. Retrieved from http://collegemathproject.senecac.on.ca/cmp/en/pdf/FinalReport /2010/ CMP%202010%20Final%20Report%20-%20English.pdf

Walle, J. A., & Lovin, L. H. (2006). *Teaching Student-Centered Mathematics (Grades 3-5*). Pearson Education, Inc.